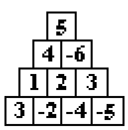
2011
MATHCOUNTS CHAPTER
SPRINT ROUND

1. A woodchuck chucks 60 pounds of wood in 1.5 days. 6 days is 4 times 1.5 days. 60 × 4 = 240 **Ans.**

2. Half the temperature of Papa's oatmeal is 20º cooler than Baby's oatmeal. Papa's oatmeal is 180º.
Let P = the temperature of Papa's oatmeal.
Let B = the temperature of Baby's oatmeal
$\frac{1}{2}$ P = B – 20
P = 180
$\frac{1}{2}$ × 180 = B – 20
90 = B – 20
B = 110 **Ans.**

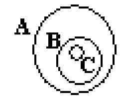
3. One number is to be selected from each of the four rows of this Number Wall. We are asked to find the largest possible product of any such four numbers.



The first and third rows (from the top) are all positive. The second and fourth rows are mixed. The largest absolute values from the second and fourth rows come from the negative numbers. Therefore, if we choose negative numbers in the second and fourth rows, we will get a positive number.
First row: 5
Second row: -6
Third row: 3
Fourth row: -5
5 × –6 × 3 × –5 =
–30 × –15 = 450 **Ans.**

4. Hannah scored 75% of April's runs. April scored 16 runs.
75% = $\frac{3}{4}$ and $\frac{3}{4}$ × 16 = 12 **Ans.**

5. Circle C has 7 items. 20 items are in Circle A but 10 of those items are not in Circle B. We are asked to find how many items are in Circle B, but not in Circle C.



Circle A has 20 items. But 10 of those items are not in Circle B. Therefore, there are 20 – 10 = 10 items in Circle B that are not in Circle A. But Circle C has 7 items. Therefore, there are 10 – 7 = 3 items that are in Circle B, but not in Circle C. 3 **Ans.**

6. A signature line is 4 in. long.
A $\frac{3}{4}$ - in. blank space is reserved at each end of the signature. So how much space is available for the signature?
$\frac{3}{4} \times 2 = \frac{6}{4} = \frac{3}{2}$
$4 - \frac{3}{2} = 2\frac{1}{2}$ **Ans.**

7. The operation $\otimes$ is defined as:
$a \otimes b = a^2 + b + 1$
$6 \otimes 5 = 6^2 + 5 + 1 =$
36 + 6 = 42 **Ans.**

8. Malton has twice as many moons as Planar. The number of Nero's moons is the cube of Malton's moons. Ufda has 4 more moons than Jir. Jir's moons are equal to double the number of Nero's moons plus the number of Planar's moons. Planar has 1 moon. How many moons does Ufda have?
Let M = the number of Malton's moons.
Let P = the number of Planar's moons.
Let N = the number of Nero's moons.
Let U = the number of Ufda's moons.
Let J = the number of Jir's moons.

We know that P = 1.
M = 2P = 2 × 1 = 2
N = $M^3 = 2^3$ = 8
J = 2N + P = 16 + 1 = 17
U = J + 4 = 17 + 4 = 21 **ANS.**

9. The sum of 3 consecutive prime numbers is 173. We are asked to find the largest of these numbers. First, let's find the mean of the 3 numbers.
3x = 173
x ≈ 58
So let's look at the primes, say between 50 and 70. They are:
53, 59, 61, 67
The one's value in 173 is 3. I think I'll try 53 + 59 + 61 since 3 + 9 + 1 = 13.
53 + 59 + 61 = 173 and the largest of the primes is 61. **Ans.**

10. $\left(3^x\right)\left(9\right) = 81$
$3^x 3^2 = 81 = 3^4$
x + 2 = 4
x = 2 **Ans.**

11. Kenton walks for 60 min. at the rate of 3 mph. He then runs for 15 min. at the rate of 8 mph. So how far does he travel?
60 min. at the rate of 3 mph is 3 miles.
15 min = $\frac{1}{4}$ hour
$\frac{1}{4}$ × 8 = 2 miles
3 + 2 = 5 **Ans.**

12. x and y are each integers greater than 3 and less than 20. What is the sum of the three possible values of x that satisfy the equation $\frac{x}{y} = \frac{3}{4}$?
Since x is greater than 3 we cannot consider $\frac{3}{4}$. The next fraction that is equivalent to $\frac{3}{4}$ is $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$.
That's a good value. Next:
$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$
That's good too. Next:
$\frac{3}{4} \times \frac{4}{4} = \frac{12}{16}$
That's also good. Are there more?
$\frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$
This one is not good because y = 20 and y < 20. Any other values will fail the requirements. Therefore, x = 6, 9, 12.
6 + 9 + 12 = 27 **Ans.**

13. We are asked to find the average number of home runs hit by the players.
6 players hit 6 home runs.
6 × 6 = 36
4 players hit 7 home runs.
4 × 7 = 28
3 players hit 8 home runs.
3 × 8 = 24
1 player hit 10 home runs.
1 × 10 = 10
There were a total of
36 + 28 + 24 + 10 = 98
home runs hit.
The number of players is
6 + 4 + 3 + 1 = 14 players.
$\frac{98}{14} = 7$ **Ans.**

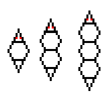
14. $\frac{5}{33} = 0.\overline{15}$
The first digit to the right of the decimal point is 1. All odd digits to the right of the decimal point are 1. The second digit to the right of the decimal point is 5. All even digits to the right of the decimal point are 5. 92 is an even number. Therefore, 5 is the value of the 92nd digit to the right of the decimal point. 5 **Ans.**

15. A player can earn either 3 points or 5 points on a turn. Capri has earned a total of 18 points. What is the fewest number of turns she could have taken?
My first guess would be 4.
5 + 5 + 5 + 3 = 18

3 + 3 + 3 + 3 + 3 + 3 = 18 but that's 6 turns.
4 **Ans.**

16. Fonks were originally priced at \$100. Then the price was increased by 20%. Finally, the price was decreased by 30%. We are asked to find the percentage of the original price that the fonk currently sells for. When the price was increased by 20% the fonk sold at 1.2 times the original price. When the price was decreased by 30% the fonk sold at 1.2 × .7 = 0.84 of the original price, or 84%. **Ans.**

17. A Growing Worm in Stage *n* contains *n* hexagons and two equilateral triangles. Because a side of the equilateral triangle is also a side of the hexagon, the sides of the hexagon are *s* and the sides of the equilateral triangle are *s*.



The perimeter of the Glowing Worm in Stage 1 is 8 and we are asked to find the perimeter of a Stage 4 Growing Worm.
In Stage 1, 4 sides of the hexagon are part of the perimeter as well as 2 sides of each equilateral triangle. That's a total of 8 sides.
8*s* = 8
s = 1
In the Stage 2 drawing, there are 4 × 2 = 8 sides of hexagons that are part of the perimeter and 2 sides of each equilateral triangle. That's a total of 8 + 4 = 12.
In the Stage 3 drawing, there are 4 × 3 = 12 sides of hexagons that are part of the perimeter and 2 sides of each equilateral triangle. That's a total of 12 + 4 = 16.
8, 12, 16. Obviously, the Stage 4 Glowing Worm has a perimeter of 20. **Ans.**

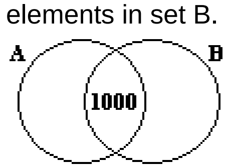
18. Each term of a sequence is one more than twice the term before it. The first term is 1. What is the sum of the first 5 terms?
The second term is (2 × 1) + 1 = 3.
The third term is (2 × 3) + 1 = 7.
The fourth term is (2 × 7) + 1 = 15.
The fifth term is (2 × 15) + 1 = 31.
31 + 15 + 7 + 3 + 1 = 57 **Ans.**

19. A fly buzzes randomly around a room 8' × 12' × 10' high. We are asked to find the probability that the fly is within 6' of the ceiling.
We can do that by dividing the volume of a room that is 8' × 12' × 6' high by the volume of the entire room.
$\frac{8\times12\times6}{8\times12\times10}=\frac{6}{10}=\frac{3}{5}$ **Ans.**

20. If 5 less than $\frac{3}{4}$ of an integer is the same as 5 more than $\frac{1}{8}$ of that integer, then what is the integer?
Let x = the integer.
$\frac{3}{4}x-5=\frac{1}{8}x+5$
$\frac{3}{4}x=\frac{1}{8}x+10$
$\frac{5}{8}x=10$
5x = 80
x = 16 **Ans.**

21. What is the sum of the negative integers that satisfy the inequality $2x-3\geq-11$?
$2x\geq-8$
$x\geq-4$
So x can be –4, –3, –2 and –1.
–4 + –3 + –2 + –1 = – 10 **Ans.**

22. The total number of elements in set A is twice the total number of elements in set B.



There are 3011 elements in the

union of A and B. The intersection of A and B is 1000 elements. We are asked to find the total number of elements in set A.
Let x + 1000 be the total number of elements in set A.
Let y + 1000 be the total number of elements in set B.
We also know that
x + y + 1000 = 3011
x + y = 2011
And, finally, we know that
2 × (y + 1000) = x + 1000
2y + 2000 = x + 1000
2y = x – 1000

$$y = \frac{x-1000}{2}$$

Substituting for y in
x + y = 2011, we get

$$x + \frac{x-1000}{2} = 2011$$

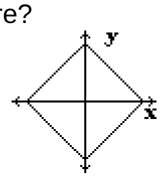
2x + x – 1000 = 4022
3x = 5022
x = 1674
1674 + 1000 = 2674 **Ans.**

23. The quotient of 2 consecutive positive integers is 1.02. So what is the sum of these 2 integers?
Since the quotient is more than 1, the numerator must be larger than the denominator.
Let x be the smaller number. Then

$$\frac{x+1}{x} = 1.02$$

x + 1 = 1.02x
1 = .02x
100 = 2x
x = 50
x + 1 = 51
50 + 51 = 101 **Ans.**

24. What is the area enclosed by the graph of $|x| + |2y| = 10$ shown here?



If x = 0, then

$$|2y| = 10$$

$$|y| = 5$$

So (0,5) and (0,-5) are the top and bottom coordinates of the area, respectively..
If y = 0 then $|x| = 10$ so
(-10,0) and (10,0) are the left and right coordinates of the area, respectively. Thus, the area is made up of 4 right triangles with legs of 5 and 10.

$$4 \times \left(\frac{1}{2} \times 5 \times 10\right) = 100$$ **Ans.**

25. Two similar right triangles have areas of 6 square inches and 150 square inches. The hypotenuse of the smaller triangle is 5 inches. We are asked to find the sum of the lengths of the legs of the larger triangle.
Clearly, the smaller triangle is a 3,4,5 right triangle.

$$\frac{1}{2} \times 3 \times 4 = 6$$

$\frac{1}{2}$ x = 150
x = 300
This is the product of the 2 legs of the larger triangle that are not the hypotenuse of the triangle.
The product of the 2 legs is 25 times the product of the two corresponding legs of the smaller triangle.
25 = 5^2 and each leg is 5 times the corresponding leg of the smaller triangle.
5 × 3 = 15
5 × 4 = 20
5 × 5 = 25
This is a 15, 20, 25 right triangle.
15 + 20 = 35 **Ans.**

26. A committee of 6 students is chosen at random from a group of 6 boys and 4 girls. What is the probability that the committee contains the same number of boys and girls?
Since the committee is composed of 6 people, to have the same number of each sex means 3 boys and 3 girls.

The number of combinations of 3 boys is

$$\frac{6!}{3!3!} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$$

The number of combinations of 3 girls is

$$\frac{4!}{3!1!} = 4$$

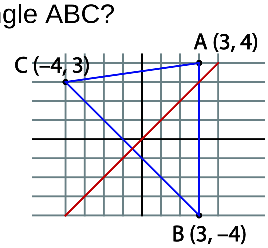
So, the number of combinations of 3 boys and 3 girls is 20 × 4 = 80 combinations.

The total number of combinations is

$$\frac{10!}{6!4!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

$$\frac{80}{210} = \frac{8}{21}$$ **Ans.**

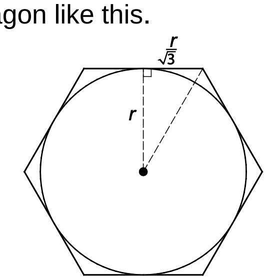
27. The point A(3,4) is reflected over the x-axis to B. B is reflected over the line y = x to C. What is the area of triangle ABC?



When a point is reflected over the x axis, its y coordinate is multiplied by -1. Therefore, B is (3,-4) and the length of AB is 8.
The red line shows the plot of y = x. When the point (a,b) is reflected over the line y = x, then the coordinates are flipped, e.g., the new point is (b,a). In this case C becomes (-4,3).
The easiest way to find the area of triangle ABC is to form a rectangle around ABC. This consists of the points A, B, (-4,4) and (-4,-4). This is a 7 × 8 rectangle so its area is 56.
The area of the triangle with points A, C, and (-4,4) is $\frac{1}{2} \times 7 \times 1 = \frac{7}{2}$
The area of the other triangle within the rectangle is formed by the points C, B, and (-4,-4). The area of this triangle is $\frac{1}{2} \times 7 \times 7 = \frac{49}{2}$
The total area of these two triangles is $\frac{7}{2} + \frac{49}{2} = \frac{56}{2} = 28$
56 – 28 = 28 **Ans.**

28. Tonisha leaves Maryville at 7:15 AM. She travels at an average speed of 45 mph. Sheila leaves an hour later averaging 60 mph. We are asked to find out at what time Sheila passes Tonisha.
Let x = the number of hours that Tonisha travels. Then
45 + 45(x – 1) = 60(x – 1)
45 + 45x – 45 = 60x – 60
45x = 60x – 60
15x = 60
x = 4
Thus, Sheila will pass Tonisha at 11:15 AM. **Ans.**

29. If Fido goes as far from the center of the yard as his leash permits, then he can create a circle. So, we have a circle inscribed in a regular hexagon like this.



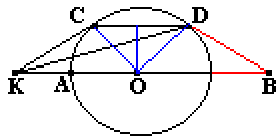
We have drawn in one of the twelve 30-60-90 triangles that exists. Letting the radius of the circle (and the long leg of the 30-60-90 triangle) be *r*, we can see that the short leg is $\frac{r}{\sqrt{3}}$, using the relationships of the 30-60-90 triangle. Therefore, Fido can reach πr^2 of his yard that measures $(12)(1/2)(\frac{r}{\sqrt{3}})(r)$. (Note that the area of the hexagon is the area of 12 of these triangles.) This is

$$\frac{\pi r^2}{(12)(1/2)(\frac{r}{\sqrt{3}})(r)} = \frac{\pi r^2}{\frac{6r^2}{\sqrt{3}}} = \frac{\sqrt{3}}{6}\pi$$

of the yard. Thus, *a* = 3, *b* = 6 and *ab* = 18. **Ans.**

30. In the figure below, circle O has radius 6 units. The length of chord CD is 8 and KA = 12 units. We are

asked to find the area of triangle KDC. The figure looks like an unfinished trapezoid and we can draw that in. (See the red lines in the figure below.)



So we will be able to find the area of triangle KDC by figuring out the area of the trapezoid CDBK and then subtracting the area of triangle DKB.
First of all, how long is KB? We know that KA = 12 and since the radius is 6, we also know that AO is 6. So KO = 18. Similarly, OB is 18 and KB = 36.
Now what is the height of the trapezoid? Using the blue lines we create triangle COD. CO = OD = 6 and CD is broken up into 2 equal line segments each of which is 4.
Let h = the height of triangle COD.
Then $4^2 + h^2 = 6^2$
$16 + h^2 = 36$ and $h^2 = 20$.
Therefore, h = $\sqrt{20} = 2\sqrt{5}$
The area of the trapezoid is
$\frac{1}{2}(8+36)2\sqrt{5} = 44\sqrt{5}$
Now we need to know the height of triangle DKB. But that's the same as the height of triangle COD or $2\sqrt{5}$. So the area of triangle DKB is $\frac{1}{2} \times 2\sqrt{5} \times 36 = 36\sqrt{5}$
Subtracting the area of triangle DKB from the area of the trapezoid gives us $44\sqrt{5} - 36\sqrt{5} = 8\sqrt{5}$ **Ans.**

TARGET ROUND

1. How many of the smallest triangular regions are there in Figure 4 of the sequence whose first three figures are shown here?

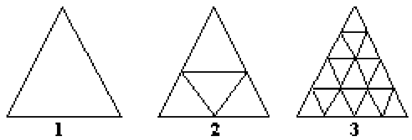


Figure 1 has 1 triangle in 1 row.
Figure 2 has 1 + 3 = 4 triangles in 2 rows.
Figure 3 has 1 + 3 + 5 + 7 = 16 triangles in 4 rows.
The number of rows doubles and each row has 2 more than the one above it.
Figure 4 must have 8 rows or
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64 triangles.
The sequence is actually
$4^0, 4^1, 4^2, 4^3, \ldots$ 64 **Ans.**

2. We have Sequence A which is
1, 5, 9, 13, …
and Sequence B which is
1, 7, 13, 19, …
We are asked to find the positive difference between the 2011th term of sequence A and the 2011th term of sequence B.
We can represent Sequence A by $a_n = 4n - 3$ where n is the number of the term in the sequence.
Therefore, the 2011th term is
$a_{2011} = 4 \times 2011 - 3 =$
$8044 - 3 = 8041$
We can represent Sequence B by $b_n = 6n - 5$ where n is the number of the term in the sequence.
Therefore, the 2011th term is
$b_{2011} = 6 \times 2011 - 5 =$
$12066 - 5 = 12061$
The positive difference is
12061 – 8041 = 4020 **Ans.**

3. The diagonal of a square is 5. The diameter of a circle is also 5. We are asked to find by how much greater the area of the square is than the area of the rectangle.
Let s = the side of the square.
Then $s^2 + s^2 = 5^2 = 25$ or
$2s^2 = 25$ and $s^2 = 12.5$
If the diameter of the circle is 5, the radius is 2.5 and the area of the circle is $\pi \times 2.5 \times 2.5 =$
$3.14159 \times 2.5 \times 2.5 \approx 19.6349375$
$19.6349375 - 12.5 \approx 7.1349375 \approx$
7.1 **Ans.**

4. Pets are either rabbits or dogs. 65% of the 840 pets participated in a parade. 180 rabbits participated and we must find out how many

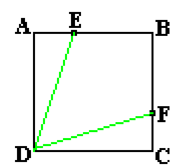
dogs participated.
840 × 0.65 = 546 pets participated.
546 – 180 = 366 pets that were not rabbits. They must be the dogs!
366 **Ans.**

5. On a scale drawing, a room measures 3.5 cm by 6 cm. 1 cm represents 1.5 m. We are asked to find the area of the room in square meters.
3.5 cm = 3.5 × 1.5 = 5.25 m
6 cm = 6 × 1.5 = 9 m
5.25 × 9 = 47.25 **Ans.**

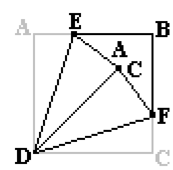
6. Each day a spider travels 5 feet up the waterspout. Each night it is washed 3 feet down the water spout. The waterspout is 50 feet long. The spider starts up the waterspout on April 1 and we are asked to find when it gets to the top.
Travelling up 5 feet and dropping 3 feet each day means that every day it actually makes 2 feet of headway.
The simplest thing is to say
$\frac{50}{2} = 25$ days.
But that would be wrong because we have to find when it will **first** reach the top. So to make sure we do it right, after 22 days (April 1 – April 22) it's 44 feet up the spout.
On April 23 it gets up to 49 feet and slips back to 46 feet.
On April 24 it gets up to 51 feet before slipping back to 48 feet. But that's enough. The answer is actually April 24!!!! **Ans.**

7. A graph shows quiz scores for a 10-question quiz. Each question is worth one point. Only 3 students answered #7 correctly but credit is given for #7 to every student. Find the new mean quiz score.
According to the graph:
3 students got 3 correct
3 students got 4 correct
5 students got 5 correct
7 students got 6 correct
5 students got 7 correct
6 students got 8 correct
4 students got 9 correct
3 students got all 10 correct – they must be the ones that answered #7 correctly, obviously.
First, figure out the number of students.
3 + 3 + 5 + 7 + 5 + 6 + 4 + 3 = 36
Now, figure out how many total points there were.
(3 × 3) + (3 × 4) + (5 × 5) + (7 × 6) + (5 × 7) + (6 × 8) + (4 × 9) + (3 × 10) = 237 points
Since there are 36 students and 3 already have #7 right, we need to add 33 points for #7 to the total.
237 + 33 = 270
$\frac{270}{36} = 7.5$ **Ans.**

8. Square ABCD has sides of length 1. Points E and F are on sides AB and CB, respectively, with AE = CF. When the square is folded along the lines DE and DF, sides AD and CD coincide and line up on diagonal BD. Given that the length of segment AE is expressed in the form $\sqrt{k} - m$, then what is the value of k + m?
Before we make the fold the square looks like this.



After the fold the square looks like this:



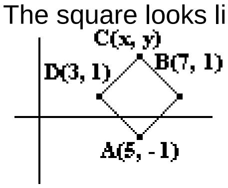
Let x = AE. The angle at DAE is 90º. DA = 1. And we know that DB, the diagonal, is $\sqrt{2}$. In the second image, if we continue the line through point A (also point C) to B, then we have the triangle EAB, where angle EAB is also 90º.
EB = 1 – x
AB = $\sqrt{2}$ – 1
Thus, we can write

$$x^2+\left(\sqrt{2}-1\right)^2=\left(1-x\right)^2$$
$$x^2+2-2\sqrt{2}+1=1-2x+x^2$$
$$3-2\sqrt{2}=1-2x$$
$$2x=1-3+2\sqrt{2}=-2+2\sqrt{2}$$
$$x=\frac{2\sqrt{2}-2}{2}=\sqrt{2}-1$$

Thus, k = 2 and m = 1.
k + m = 2 + 1 = 3 **Ans.**

TEAM ROUND

1. 3 vertices of square ABCD are located at A(5, -1), B(7, 1) and D(3,1). We are asked to find the coordinates of point C.
   The square looks like this:



   The difference between the x coordinate of B and the x coordinate of A is 7 – 5 = 2.
   The difference between the y coordinate of B and the y coordinate of A is 1 – (–1) = 2.
   The coordinates of D are (3, 1)
   If x is the coordinate of point C, then x = 3 + 2 = 5.
   If y is the coordinate of point C, then y = 1 + 2 = 3.
   Therefore, the coordinates of point C are (5, 3). **Ans.**

2. What is the units digit of 3^{2011}?
   If you look at the units digits of the powers of 3, you see a pattern.
   3, **9**, 2**7**, 8**1**, 24**3**, 72**9**, 218**7**, 656**1** or 3, 9, 7, 1.
   When the power is divisible by 4 and has a remainder of 1, then the units digit is 3. If it's divisible by 4 and has a remainder of 2, then the units digit is 9. If the remainder is 3, then the units digit is 7 and if the remainder is 0, then the units digit is 1. All we have to do is divide 2011 by 4.
   $\frac{2011}{4}=502\frac{3}{4}$ so the remainder is 3. Therefore the units digit is 7. **Ans.**

3. The sale price that Mr. Adams paid for a 10-ft by 12-ft piece of carpet was the same as the non-sale price of a piece of carpet measuring 6-ft by 8-ft. We are asked to find the percent off the original carpet price that was taken to create the sale price.
   Let x = the regular price.
   Let y = the sale price.
   Then (6 × 8)x = (10 × 12)y
   48x = 120y
   $\frac{y}{x}=\frac{48}{120}=\frac{4}{10}$
   So y, the sale price, was 40% of x, the original price. Therefore, the sale price was reduced by 100 – 40 = 60% **Ans.**

4. Zeta runs around a track at a rate of 30 laps per 75 min. Ray runs around the track at a rate of 20 laps per 40 min. We must find how many minutes it takes them to run a combined distance of 99 laps.
   30 laps per 75 minutes means that Zeta runs $\frac{30}{75}=\frac{6}{15}=\frac{2}{5}$ lap each minute.
   20 laps per 40 minutes means that Ray runs $\frac{20}{40}=\frac{1}{2}$ lap each minute.
   Together they run $\frac{2}{5}+\frac{1}{2}=\frac{4}{10}+\frac{5}{10}=\frac{9}{10}$ lap each minute.
   $\frac{99}{\frac{9}{10}}=99\times\frac{10}{9}=11\times10=110$ minutes. 110 **Ans.**

5. Line L passes through the points $\left(0,\frac{1}{2}\right)$ and $\left(4,k\right)$. Line L is perpendicular to the line

y = –4x + 5. We are asked to find the value of k.
First, let's come up with line L.
y = mx + b

$$\frac{1}{2} = 0m + b$$

$$b = \frac{1}{2}$$

Using $(4, k)$,

$$k = 4m + \frac{1}{2}$$

$$2k = 8m + 1$$

$$8m = 2k - 1$$

$$m = \frac{2k-1}{8}$$

We know that the slope of a line perpendicular to line L must be $-\frac{1}{m}$ and we know that the slope of the perpendicular line is –4.
Therefore,

$$-\frac{1}{m} = -\frac{1}{\frac{2k-1}{8}} = -4$$

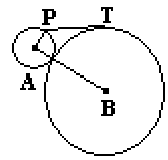
$$\frac{1}{\frac{2k-1}{8}} = 4$$

$$\frac{8}{2k-1} = 4$$

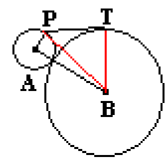
8 = 8k – 4
8k = 12
k = $\frac{12}{8} = \frac{3}{2}$ **Ans.**

6. Movie showings begin at 11:15 a.m. Each showing consists of 10 minutes of previews and 105 minutes of the movie. It takes 20 minutes to get the theater ready for the next showing and there are 5 showings of the movie prior to midnight. We are asked to find the earliest possible time for the last showing to begin.
Since there are 5 showings, the last showing begins after 4 sequences of previews, movie and cleanup occur.
10 + 105 + 20 = 135 minutes.
135 × 4 = 540 minutes
$\frac{540}{60} = 9$ hours
11:15 a.m. + 9 hours =
8:15 p.m. **Ans.**

7. Circles A and B are externally tangent. Angle PAB is a right angle. Segment PT is tangent to circle B at T. The radius of circle A is 1 cm and the radius of circle B is 7 cm. We are asked to find the length of segment PT.



Let's draw lines from B to T and B to P.



Angle PAB and Angle PTB are 90º angles. We know that the length of AP is 1 and the length of AB is 1 + 7 = 8. Therefore, the length of PB is:

$$\sqrt{1^2 + 8^2} = \sqrt{65}$$

The length of TB is 7. Let x = the length of PT. Then

$$x^2 + 7^2 = \sqrt{65}^2$$

$$x^2 + 49 = 65$$

$$x^2 = 65 - 49 = 16$$

x = 4 **Ans.**

8. Originally, the first digit in a 3-digit area code could not be a 0 or 1. The second digit could only be a 0 or 1. The third digit could be anything. The restrictions on the second digit were lifted and we are asked to determine how many more 3-digit area codes are possible now. Before the restrictions were lifted, we could have 8 possibilities for the

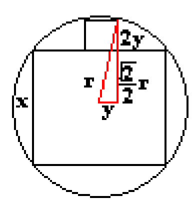
first digit, 2 for the second and 10 for the third. That's a total of
8 × 2 × 10 possibilities.
Now, instead of 2 for the middle digit we have 10 possibilities or
8 × 10 × 10 possibilities.
8 × 10 × 10 – (8 × 2 × 10) =
(8 × 10) × (10 – 2) =
80 × 8 = 640 **Ans.**

9. Each of the nine digits {1, 2, 3,…, 9} is used exactly once as a digit in either a four-digit positive integer *a* or the five-digit positive integer *b*. We must find the smallest possible value of *a* if

$$\frac{a}{b} = \frac{1}{2}$$

So we know that *b* = 2*a* and that means *b* must end in 2, 4, 6, or 8.
How big can *b* be?
9876 × 2 =19752. Therefore, since *b* is a 5-digit integer, the ten-thousandth's digit must be 1.
What is the smallest 4-digit number that multiplied by 2 gives a 5-digit number? That must be 5000 because 5000 × 2 = 10000.
So the thousandth's digit of $a \geq 5$.
And since 1 is now reserved and 0 cannot be used, we have to result in a number that is $\geq$ 12000. So $a \geq$ 6000.
Let us reserve 6, 1 and 2. That leaves 3, 4, 5, 7, 8, 9 for the smallest value of *a*. 5 cannot be in the one's column because 5 × 2 = 10. Neither can 3 because 3 × 2 = 6 or 8 because 8 × 2 = 16.
6300 × 2 = 12600 which uses 6 twice. Therefore, we have to start at 6350 × 2 = 12700.
Is there anything else we can remove before trying this?
If the one's column of *a* is 4, then 8 cannot be in *a*.
If the one's column of *a* is 7, then 4 cannot be in *a*.
If the one's column of *a* is 9, then 8 cannot be in *a*. Neither can the last two digits of *a* be between 50 and 60 because, at minimum, that will make the ten's column of *b* be 0 or 1. So, we start at 6350 but immediately move up to 6360 which we can't do and then up to 6370.
Can't use 1, 2, 3 in the one's column.
6374 × 2 = 12748 No good.
6379 × 2 = 12758 No good.
6380 × 2 = 12760 so we have to look greater than 6384.
6387 × 2 = 12774 No good.
6389 × 2 = 12778 No good.
6394 × 2 = 12788 No good.
6397 × 2 = 12794 No good.
So we're now looking at 6400 and up.
Remember, 0, 1, and 2 are out for the ten's column.
6430 × 2 = 12860 so it must be greater than 6434.
6439 × 2 = 12878 No good.
6479 × 2 = 12958 No good.
6480 – 6484 is out.
6489 × 2 = 12978 No good.
Nothing in the 6490's will work either. Where does that leave us?
6500 × 2 = 13000
That means that 1, 3, and 6 are now reserved. But 6500 – 6549 are out because there will be a 0 in *b*.
So are 6550-6599 because there will be a 1 in *b.*
Next up is 6600 but that's 2 6's.
So now we're at 6700 × 2 = 13400.
6729 × 2 = 13458 !!!!! **Ans.**

10. A square is inscribed in a circle and there is a smaller square with one side coinciding with a side of the larger square and two vertices on the circle. We are asked to find the percentage of the larger square's area that the smaller square's area has.



Let x = the side of the larger square.
Let y = half the side of the smaller square.
Let r = the radius of the circle.
The diagonal of the larger square is 2r. Find the area of the larger square.

$$x^2 + x^2 = (2r)^2$$
$$2x^2 = 4r^2$$
$$x^2 = 2r^2$$
$$x = \sqrt{2}r$$
The area of the smaller square is
$$(2y)^2 = 4y^2$$
The ratio of the area of the smaller square to the area of the larger square is:
$$\frac{4y^2}{x^2} = \frac{4y^2}{2r^2} = 2\frac{y^2}{r^2}$$
We can construct a triangle whose legs are y and $\frac{\sqrt{2}}{2}r + 2y$. The hypotenuse of this right triangle is r.
$$y^2 + \left(\frac{\sqrt{2}}{2}r + 2y\right)^2 = r^2$$
$$y^2 + \frac{1}{2}r^2 + 2\left(2y \times \frac{\sqrt{2}}{2}r\right) + 4y^2 =$$
$$5y^2 + 2\sqrt{2}ry + \frac{1}{2}r^2 = r^2$$
Divide by r^2.
$$5\frac{y^2}{r^2} + 2\sqrt{2}\frac{y}{r} + \frac{1}{2} = 1$$
$$5\frac{y^2}{r^2} + 2\sqrt{2}\frac{y}{r} - \frac{1}{2} = 0$$
Remember that we would really like to find out what $2\frac{y^2}{r^2}$ is so define z = $\frac{y}{r}$ and rewrite the equation.
$$5z^2 + 2\sqrt{2}z - \frac{1}{2} = 0$$
$$10z^2 + 4\sqrt{2}z - 1 = 0$$
This can be solved using the quadratic formula
$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a = 10$, $b = 4\sqrt{2}$ and $c = -1$.
$$z = \frac{-4\sqrt{2} \pm \sqrt{(4\sqrt{2})^2 - 4 \times 10 \times -1}}{20}$$
$$z = \frac{-4\sqrt{2} \pm \sqrt{32 + 40}}{20}$$
$$z = \frac{-4\sqrt{2} \pm \sqrt{72}}{20} = \frac{-4\sqrt{2} \pm 6\sqrt{2}}{20}$$
Remember that z must be positive.
$$z = \frac{y}{r} = \frac{2\sqrt{2}}{20} = \frac{\sqrt{2}}{10}$$
$$2\frac{y^2}{r^2} = 2\left(\frac{\sqrt{2}}{10}\right)^2 = 2 \times \frac{2}{100} = \frac{4}{100}$$
$\frac{4}{100} = 4\%$ **Ans.**